Univerzitet u Zenici
Ekonomski fakultet
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Parcijalni ispit iz predmeta Matematika

GRUPA A

1. Dokazati matematičkom indukcijom da važi: $100 \mid 7 + 7^2 + 7^3 + \ldots + 7^{4n}$, $n \in \mathbb{N}$.
2. Odrediti sve kompleksne brojeve z čiji je argument $\arg(z) = \dfrac{5\pi}{4}$ i $|z+2| = 3$.
3. Riješiti sistem jednačina i diskutovati rješenja sistema u zavisnosti od parametra:
$$(a+1)x + 2y + 3z = 0$$
$$x + (a+2)y + 3z = 0$$
$$x + 2y + (a+3)z = 0.$$
4. Izračunati limese: $L_1 = \lim\limits_{n\to\infty}\left[\dfrac{(n+1)^6 - (n-2)^6}{3n^5 + 2n^4 - n^3 + 1}\right]$ i $L_2 = \lim\limits_{n\to\infty} n\left(1 - \sqrt[3]{1 - \dfrac{1}{n}}\right)$.

GRUPA B

1. Izračunati x ako je četvrti član u razvoju binoma $\left(10^{\log\sqrt{x}} + \dfrac{1}{\sqrt[\log x]{10}}\right)^7$ jednak 3500000.
2. Napisati brojeve $z_1 = 1 + \cos\alpha + i\sin\alpha$, $z_2 = 1 + \cos\alpha - i\sin\alpha$, $z = \dfrac{z_1}{z_2}$ u trigonometrijskom obliku. Razlikovati dva slučaja: $\alpha \in (0, \pi)$ i $\alpha \in (\pi, 2\pi)$.
3. Riješiti sistem jednačina i diskutovati rješenja sistema u zavisnosti od parametra:
$$x + (2+m)y - z = 0$$
$$(2+m)x + y - z = 1$$
$$x + y - (2+m)z = m + 3.$$
4. Izračunati limese: $L_1 = \lim\limits_{n\to\infty}\dfrac{(1-n)^4 + (2-n)^4}{n\left[(1-n)^3 - (1+n)^3\right]}$ i $L_2 = \lim\limits_{n\to\infty}\left(\sqrt{n + \sqrt{n + \sqrt{n}}} - \sqrt{n}\right)$.

GRUPA C

1. Dokazati matematičkom indukcijom da važi: $10 \mid (2n^5 + 5n^4 + 3n),\ n \in \mathbb{N}$.
2. Izračunati sve vrijednosti korjena $\sqrt[4]{z}$, ako je
$$z = \left(2 + i\sqrt{12}\right)(1-i)\left(2\sqrt{2}\cos\frac{11\pi}{12} + i\sqrt{8}\sin\frac{11\pi}{12}\right).$$
3. Odrediti vrijednost parametra k tako da vektori $\mathbf{a} = (k, 1-k, k)$, $\mathbf{b} = (2k, 2k-1, k+2)$, $\mathbf{c} = (-2k, k, -k)$ budu linearno zavisni i za najveću dobijenu cjelobrojnu vrijednost parametra k, napisati vektor **b** kao linearnu kombinaciju vektora **a** i **c**.
4. Izračunati limese: $L_1 = \lim\limits_{n\to\infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}$ i $L_2 = \lim\limits_{n\to\infty}\left(n\sqrt[3]{2} - \sqrt[3]{2n^3 + 5n^2 - 7}\right)$.

GRUPA D

1. Za koje x je četvrti član razvoja binoma $\left(\sqrt{2^{x-1}} + \frac{1}{\sqrt[3]{2^x}}\right)^n$ jednak $20n$, ako je binomni koeficijent četvrtog člana pet puta veći od binomnog koeficijenta drugog člana.
2. Riješiti jednačinu $z^4 + 2z^2 + 4 = 0$ u skupu kompleksnih brojeva.
3. Diskutovati rang matrice u zavisnosti od parametra $A = \begin{bmatrix} 8-\lambda & 2 & 3 & \lambda \\ 1 & 9-\lambda & 4 & \lambda \\ 1 & 2 & 10-\lambda & \lambda \\ 1 & 2 & 3 & \lambda \end{bmatrix}$.
4. Izračunati limese: $L_1 = \lim\limits_{n\to\infty} \frac{(n+3)!}{(n+2)! + (n+4)!}$ i $L_2 = \lim\limits_{n\to\infty}\left(\sqrt[4]{n^4 + 4n^3 + 4n^2 + 4n + 1} - n\right)$.

Neki zadaci nisu detaljno raspisani.

Grupa A E.F. I parcijalni

1. $n=1 \Rightarrow 100 \mid 7+7^2+7^3+7^4$

$100 \mid 2800$ očito vrijedi

P: $100 \mid 7+7^2+\dots+7^{4k} \Rightarrow 7+7^2+\dots+7^{4k}=100m,\ m\in\mathbb{N}$

T: $100 \mid \underbrace{7+7^2+\dots+7^{4k}}_{100m}+7^{4k+1}+7^{4k+2}+7^{4k+3}+7^{4k+4}$

$=100m+7^{4k}\cdot(7+7^2+7^3+7^4)=100m+7^{4k}\cdot 2800$

$=100(m+7^{4k}\cdot 28)$ – tvrdnja je tačna i za $n=k+1$.

2. $\arg(z)=\frac{5\pi}{4} \Rightarrow z=\rho\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$

$z=\rho\left(-\frac{\sqrt{2}}{2}-i\cdot\frac{\sqrt{2}}{2}\right)$

$\Rightarrow z+2=-\frac{\sqrt{2}}{2}\rho+2-i\cdot\frac{\sqrt{2}}{2}$

$|z+2|=\sqrt{\left(-\frac{\sqrt{2}}{2}\rho+2\right)^2+\left(-\frac{\sqrt{2}}{2}\right)^2}=3 \quad /^2$

$4-2\cdot 2\cdot\frac{\rho\sqrt{2}}{2}+\frac{2}{4}\rho^2+\frac{2}{4}=9$

$4-2\rho\sqrt{2}+\frac{1}{2}\rho^2+\frac{1}{2}=9 \quad /\cdot 2$

$8-4\rho\sqrt{2}+\rho^2+1=18$

$\rho^2-4\rho\sqrt{2}-9=0$

$D=16\cdot 2+36=68=\cancel{36}\ 4\cdot 17$

$\rho_{1,2}=\frac{4\sqrt{2}\pm 2\sqrt{17}}{2}=2\sqrt{2}\pm\sqrt{17}$

Pošto je $\rho>0$ zaključujemo: $\rho=2\sqrt{2}+\sqrt{17}$

$z=(2\sqrt{2}+\sqrt{17})\cdot\left(\frac{\sqrt{2}}{2}+i\cdot\frac{\sqrt{2}}{2}\right)$

$z=2+\frac{\sqrt{34}}{2}+i\cdot\left(2+\frac{\sqrt{34}}{2}\right)$

$z=\left(2+\frac{\sqrt{34}}{2}\right)(1+i)$

3. Sistem je homogeni

$$D = \begin{vmatrix} a+1 & 2 & 3 \\ 1 & a+2 & 3 \\ 1 & 2 & a+3 \end{vmatrix} \overset{I_k + II_k + III_k}{=}$$

$$= \begin{vmatrix} a+6 & 2 & 3 \\ a+6 & a+2 & 3 \\ a+6 & 2 & a+3 \end{vmatrix} = (a+6) \begin{vmatrix} 1 & 2 & 3 \\ 1 & a+2 & 3 \\ 1 & 2 & a+3 \end{vmatrix}$$

$$= (a+6) \cdot \begin{vmatrix} 1 & 2 & 3 \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = a^2(a+6)$$

$D = 0 \Rightarrow a = 0 \lor a = -6$

1° $a \neq 0 \land a \neq -6 \Rightarrow$ sistem ima samo jedno (trivijalno) rješenje $(0, 0, 0)$

2° $a = 0 \Rightarrow$

$$x + 2y + 3z = 0$$
$$x + 2y + 3z = 0$$
$$x + 2y + 3z = 0$$

$y = \alpha$, $z = \beta$ – proizvoljne konstante

$x = -2\alpha - 3\beta$

Rješenja $(-2\alpha - 3\beta, \alpha, \beta)$

3° $a = -6 \Rightarrow$

$$-5x + 2y + 3z = 0 \quad \ldots (1)$$
$$x - 4y + 3z = 0 \quad \ldots (2)$$
$$x + 2y - 3z = 0 \quad \ldots (3)$$

$(1) - (2)$: $-6x + 6y = 0 \Rightarrow x = y$

$(2) + (3)$: $2x - 2y = 0 \Rightarrow x = y$

$(3) \Rightarrow y + 2y - 3z = 0 \Rightarrow 3y - 3z = 0 \Rightarrow y = z$

Ryesegja: (α, α, α), $\alpha = \text{const.}$

4.

$$L_1 = \lim_{n\to\infty} \frac{(n+1)^6 - (n-2)^6}{3n^5+2n^4-n^3+1} =$$

$$= \lim_{n\to\infty} \frac{n^6 + 6n^5 + 15n^4 + 20n^3 + 15n^2 + 6n + 1 - (n^6 - 6\cdot 2n^5 + 15\cdot n^4\cdot 2^2 - \dots}{3n^5+2n^4-n^3+1}$$

$$= \lim_{n\to\infty} \frac{6n^5 + 6\cdot 2n^5}{3n^5} = \frac{18}{3} = 6$$

$$L_2 = \lim_{n\to\infty} n\cdot\left(1 - \sqrt[3]{1-\frac{1}{n}}\right)\cdot \frac{1+\sqrt[3]{1-\frac{1}{n}} + \sqrt[3]{\left(1-\frac{1}{n}\right)^2}}{1+\sqrt[3]{1-\frac{1}{n}} + \sqrt[3]{\left(1-\frac{1}{n}\right)^2}}$$

$$= \lim_{n\to\infty} \frac{n\cdot\left(1 - \sqrt[3]{\left(1-\frac{1}{n}\right)^3}\right)}{1+\sqrt[3]{1-\frac{1}{n}} + \sqrt[3]{1-\left(\frac{1}{n}\right)^2}}$$

$$= \lim_{n\to\infty} \frac{n\cdot\left(1-1+\frac{1}{n}\right)}{3} = \frac{1}{3}$$

Grupa B, I parc. E.K

1. $T_4 = 35 \cdot 10^5$

$T_4 = \binom{7}{3}\left(10^{\log\sqrt{x}}\right)^4 \cdot \left(\frac{1}{10^{\frac{1}{\log x}}}\right)^3 = 35 \cdot 10^5$

$\frac{7\cdot 6\cdot 5}{1\cdot 2\cdot 3} \cdot (\sqrt{x})^4 \cdot 10^{-\frac{3}{\log x}} = 35 \cdot 10^5 \ /:35$

$x^2 \cdot 10^{-\frac{3}{\log x}} = 10^5$

$\log x^2 \cdot 10^{-3/\log x} = \log 10^5$

$\log x^2 + \log 10^{-\frac{3}{\log x}} = 5$

$2\log x - \frac{3}{\log x} = 5, \quad \log x = t, \quad x > 0$

$2t - \frac{3}{t} = 5 \quad / \cdot t$

$2t^2 - 5t - 3 = 0$

$D = 25 + 24 = 49$

$t_{1,2} = \frac{5 \pm 7}{4} \Rightarrow t_1 = 3, \quad t_2 = -\frac{1}{2}$

$\log x = 3 \Rightarrow x_1 = 10^3 = 1000$

$\log x = -\frac{1}{2} \Rightarrow x_2 = 10^{-1/2} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

2. $z_1 = 1 + \cos\alpha + i\sin\alpha$

$z_1 = 2\cos^2\frac{\alpha}{2} + i \cdot 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$z_1 = 2\cos\frac{\alpha}{2} \cdot \left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)$

Ako je $\alpha \in (0, \pi)$ tada $\frac{\alpha}{2} \in \left(0, \frac{\pi}{2}\right) \Rightarrow 2\cos\frac{\alpha}{2} > 0$

pa je z_1 napisano u trig. obliku.

Ako je $\alpha \in (\pi, 2\pi)$, tada $\frac{\alpha}{2} \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow 2\cos\frac{\alpha}{2} < 0$

$\Rightarrow z_1 = -2\cos\frac{\alpha}{2}\left(-\cos\frac{\alpha}{2} - i\sin\frac{\alpha}{2}\right)$

$$z_1 = -2\cos\frac{\alpha}{2}\left[\cos\left(\pi+\frac{\alpha}{2}\right) + i\sin\left(\pi+\frac{\alpha}{2}\right)\right]$$

$$z_2 = 1+\cos\alpha - i\sin\alpha = 2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2} - i\sin\frac{\alpha}{2}\right)$$

$$\alpha\in(0,\pi) \Rightarrow z_2 = 2\cos\frac{\alpha}{2}\cdot\left[\cos\left(-\frac{\alpha}{2}\right) + i\sin\left(-\frac{\alpha}{2}\right)\right]$$

$$\alpha\in(\pi,2\pi) \Rightarrow z_2 = -2\cos\frac{\alpha}{2}\left(-\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)$$

$$= -2\cos\frac{\alpha}{2}\left[\cos\left(\pi-\frac{\alpha}{2}\right) + i\sin\left(\pi-\frac{\alpha}{2}\right)\right]$$

$$z = \frac{z_1}{z_2} = ?$$

1° $\alpha\in(0,\pi) \Rightarrow z = \dfrac{2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)}{2\cos\frac{\alpha}{2}\left[\cos\left(-\frac{\alpha}{2}\right) + i\sin\left(-\frac{\alpha}{2}\right)\right]}$

$$z = \cos\left(\alpha+\frac{\alpha}{2}\right) + i\sin\left(\alpha+\frac{\alpha}{2}\right)$$

$$z = \cos\frac{3\alpha}{2} + i\sin\frac{3\alpha}{2}$$

2° $\alpha\in(\pi,2\pi) \Rightarrow z = \dfrac{-2\cos\frac{\alpha}{2}\left[\cos\left(\pi+\frac{\alpha}{2}\right) + i\sin\left(\pi+\frac{\alpha}{2}\right)\right]}{-2\cos\frac{\alpha}{2}\left[\cos\left(\pi-\frac{\alpha}{2}\right) + i\sin\left(\pi-\frac{\alpha}{2}\right)\right]}$

$$z = \cos\left(\pi+\frac{\alpha}{2}-\pi+\frac{\alpha}{2}\right) + i\sin\left(\pi+\frac{\alpha}{2}-\pi+\frac{\alpha}{2}\right)$$

$$z = \cos\alpha + i\sin\alpha$$

3.

$$D = (m+1)^2(m+4)$$

$$D_x = 0$$

$$D_y = -(m+1)(m+4)$$

$$D_z = -(m+1)(m+2)(m+4)$$

1° $m\neq -1 \wedge m\neq -4 \Rightarrow$ tačno jedno rješenje

$$\left(0, \frac{-1}{m+1}, \frac{-m-2}{m+1}\right)$$

2° $m=-1 \Rightarrow$ sistem nema rješenja

3° $m = 4 \Rightarrow D = D_x = D_y = D_z = 0$

opšte glavni: $x - 2y + z = 0$

$-2x + y + z = -1$

$x + y - 2z = -1$

$$X = \frac{3\alpha - 1}{3}$$

$y = \alpha$, $z = \frac{3\alpha - 1}{3}$

Rješenja: $\left(\frac{3\alpha - 1}{3}, \alpha, \frac{3\alpha - 1}{3}\right)$ – beskonačno mnogo rješenja

4. $$L_1 = \lim_{n \to \infty} \frac{(1-n)^4 + (2-n)^4}{n \cdot \left[(1-n)^3 - (1+n)^3\right]} =$$

$$= \lim_{n \to \infty} \frac{1 - 4n + 6n^2 - 4n^3 + n^4 + 2^4 - 4 \cdot 2^3 n + 6 \cdot 2^2 n^2 - 4 \cdot 2 n^3 + n^4}{n \cdot \left[1 - 3n + 3n^2 - n^3 - (1 + 3n + 3n^2 + n^3)\right]}$$

$$= \lim_{n \to \infty} \frac{17 - 12n + 30n^2 - 12n^3 + 2n^4}{n \cdot (-2n^3 - 6n)}$$

$$= \lim_{n \to \infty} \frac{2n^4}{-2n^4} = -1$$

$$L_2 = \lim_{n \to \infty} \left(\sqrt{n + \sqrt{n + \sqrt{n}}} - \sqrt{n}\right) \cdot \frac{\sqrt{n + \sqrt{n + \sqrt{n}}} + \sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}} + \sqrt{n}}$$

$$= \lim_{n \to \infty} \frac{n + \sqrt{n + \sqrt{n}} - n}{\sqrt{n + \sqrt{n + \sqrt{n}}} + \sqrt{n}} \quad \begin{matrix} /:\sqrt{n} \\ /:\sqrt{n} \end{matrix}$$

$$= \lim_{n \to \infty} \frac{\sqrt{1 + \frac{\sqrt{n}}{n}}}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{n}} + 1} = \frac{1}{2}$$

Grupa C, I parcijalni E.T.

1. $10 \mid (2n^5 + 5n^4 + 3n)$, $n \in N$

$n = 1 \Rightarrow 10 \mid 10$

$P: 2k^5 + 5k^4 + 3k = 10m$, $m \in N$

$n = k+1 \Rightarrow 2(k+1)^5 + 5(k+1)^4 + 3(k+1) = 10t$, $t \in N$

$2(k+1)^5 + 5(k+1)^4 + 3(k+1) =$

$= 2(k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1) + 5(k^4 + 4k^3 + 6k^2 + 4k + 1) + 3k + 3$

$= \underbrace{(2k^5 + 5k^4 + 3k)}_{10m \text{ po pretpostavci}} + 10(k^4 + 2k^3 + 2k^2 + k + 2k^3 + 3k^2 + 2k + 1)$

$= 10(m + k^4 + 4k^3 + 5k^2 + 3k + 1)$

2. $2 + i\sqrt{12} = 2 + i\cdot\sqrt{4\cdot 3} = 2 + 2i\sqrt{3} = 2(1 + i\sqrt{3})$

$= 4\cdot\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$1 - i = \sqrt{2}\cdot\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ $\quad 4\cdot\sqrt{2}\cdot 2\sqrt{2}$

$\Rightarrow z = 4\cdot\sqrt{2}\cdot\sqrt{8}\left[\cos\left(\frac{\pi}{3} + \frac{7\pi}{4} + \frac{11\pi}{12}\right) + i\sin\left(\frac{\pi}{3} + \frac{7\pi}{4} + \frac{11\pi}{12}\right)\right]$

$= 16\cdot\left(\cos\frac{4\pi + 21\pi + 11\pi}{12} + i\sin\frac{4\pi + 21\pi + 11\pi}{12}\right)$

$= 16\cdot(\cos 3\pi + i\sin 3\pi)$, $3\pi = \underbrace{2\pi}_{\text{period}} + \pi$ $\quad \sqrt[4]{16} = 2$

$z = 16\cdot(\cos\pi + i\sin\pi)$

$\sqrt[4]{z} = \sqrt[4]{16}\cdot\left(\cos\frac{\pi + 2k\pi}{4} + i\sin\frac{\pi + 2k\pi}{4}\right)$, $k = 0, 1, 2, 3$

$k = 0 \Rightarrow \sqrt[4]{z} = 2\cdot\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \sqrt{2} + i\sqrt{2}$

$k = 1 \Rightarrow \sqrt[4]{z} = 2\left(-\frac{\sqrt{2}}{2} + i\cdot\frac{\sqrt{2}}{2}\right) = -\sqrt{2} + i\sqrt{2}$

$k = 2 \Rightarrow \sqrt[4]{z} = 2\left(-\frac{\sqrt{2}}{2} - i\cdot\frac{\sqrt{2}}{2}\right) = -\sqrt{2} - i\sqrt{2}$

$k = 3 \Rightarrow \sqrt[4]{z} = 2\left(\frac{\sqrt{2}}{2} - i\cdot\frac{\sqrt{2}}{2}\right) = \sqrt{2} - i\sqrt{2}$

3.

$$\begin{vmatrix} k & 1-k & k \\ 2k & 2k-1 & k+2 \\ -2k & k & -k \end{vmatrix} = 0 \quad \begin{pmatrix} I_k + 2\cdot II_k \\ III_k + I_k \end{pmatrix} \Rightarrow$$

$$\begin{vmatrix} 2-k & 1-k & 1 \\ 6k-2 & 2k-1 & 3k+1 \\ 0 & k & 0 \end{vmatrix} = 0$$

$$-k\cdot\left[(2-k)(3k+1)-(6k-2)\right]=0$$

$$-k\cdot(6k+2-3k^2-k-6k+2)=0$$

$$-k\cdot(-3k^2-k+4)=0 \quad /\cdot(-1)$$

$$k(3k^2+k-4)=0$$

$$k_1=0, \quad 3k^2+k-4=0 \Rightarrow k_2=1, \quad k_3=-\frac{4}{3}$$

$$k=1 \Rightarrow a=(1,0,1),\ b=(2,1,3),\ c=(-2,1,-1)$$

$$b=\alpha a+\beta c$$

$$(2,1,3)=\alpha(1,0,1)+\beta(-2,1,-1)$$

$$(2,1,3)=(\alpha-2\beta,\ \beta,\ \alpha-\beta)$$

$$\Rightarrow \begin{cases} \alpha-2\beta=2 \\ \beta=1 \\ \alpha-\beta=3 \end{cases} \Rightarrow \alpha=4,\ \beta=1 \Rightarrow \boxed{b=4a+c}$$

(4) $\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!} = \lim_{n\to\infty} \frac{(n+1)!\,(n+2+1)}{(n+1)!\,(n+2-1)} = 1$

$\lim_{n\to\infty} \left(n\cdot\sqrt[3]{2}-\sqrt[3]{2n^3+5n^2-7}\right)\cdot \frac{(n\sqrt[3]{2})^2+\sqrt[3]{2n^3+5n^2-7}\cdot n\sqrt[3]{2}+\sqrt[3]{\ldots}}{\ldots}$

$= \lim_{n\to\infty} \frac{2n^3-(2n^3+5n^2-7)}{n^2\cdot\sqrt[3]{4}+n\cdot\sqrt[3]{2}\cdot\sqrt[3]{2n^3+\ldots}+\sqrt[3]{(2n^3+\ldots)^2}} =$

$= -\frac{5}{3\cdot\sqrt[3]{4}}$

Grupa D I. ponavljanje 2.F

1. $\binom{n}{3} = 5 \cdot \binom{n}{1}$

$\frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} = 5n \quad / \cdot \frac{6}{n}$

$(n-1)(n-2) = 30$

$n^2 - 2n - n + 2 - 30 = 0$

$n^2 - 3n - 28 = 0 \Rightarrow \boxed{n = 7} \vee n = -4$ → ne odgovara prirodi zadatka

$\begin{cases} T_4 = 20 \cdot 7 = 140 \\ T_4 = \binom{7}{3} \left(\sqrt{2^{x-1}}\right)^4 \cdot \left(\frac{1}{\sqrt[3]{2^x}}\right)^3 \end{cases}$

$\Rightarrow \frac{7\cdot 6\cdot 5}{1\cdot 2\cdot 3} \cdot \left(2^{x-1}\right)^2 \cdot \frac{1}{2^x} = 140 \quad / : 35$

$2^{2x-2-x} = 4 = 2^2$

$x - 2 = 2 \Rightarrow \boxed{x = 4}$

2.

$z^4 + 2z^2 + 4 = 0$

smjena: $z^2 = t \Rightarrow t^2 + 2t + 4 = 0$

$t_{1,2} = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm \sqrt{4\cdot 3\cdot(-1)}}{2}$

$t_{1,2} = \frac{-2 \pm 2\sqrt{3}i}{2} = -\frac{2}{2} \pm \frac{2\sqrt{3}}{2} i$

$t_{1,2} = -1 \pm \sqrt{3} i$

$z^2 = t \Rightarrow z = \sqrt{t}$

$z_{1,2} = \sqrt{-1+\sqrt{3}i}$, $z_{3,4} = \sqrt{-1-\sqrt{3}i}$

$-1 + \sqrt{3}i = 2 \cdot \left(\cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}\right)$

$$\sqrt{2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)}=\sqrt{2}\cdot\left(\cos\underbrace{\frac{\frac{2\pi}{3}+2k\pi}{2}}_{\frac{\pi}{3}+k\pi}+i\sin\underbrace{\frac{\frac{2\pi}{3}+2k\pi}{2}}_{\frac{\pi}{3}+k\pi}\right)$$

a pri tome je $k=0,1$

$$k=0 \Rightarrow z_1=\sqrt{2}\cdot\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$

$$z_1=\sqrt{2}\cdot\left(\frac{1}{2}+i\cdot\frac{\sqrt{3}}{2}\right)$$

$$z_1=\frac{\sqrt{2}}{2}+i\frac{\sqrt{6}}{2}$$

$$k=1 \Rightarrow z_2=\sqrt{2}\cdot\left[\cos\left(\frac{\pi}{3}+\pi\right)+i\sin\left(\frac{\pi}{3}+\pi\right)\right]$$

$$=-\left(\frac{\sqrt{2}}{2}+i\cdot\frac{\sqrt{6}}{2}\right)$$

~~$k=2 \Rightarrow z_3=\sqrt{2}\cdot\left[\cos\frac{4\pi}{3}+\right.$~~

$$-1-i\sqrt{3}=2\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$$

$$\sqrt{2\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)}=\sqrt{2}\left(\cos\frac{\frac{4\pi}{3}+2k\pi}{2}+i\sin\frac{\frac{4\pi}{3}+2k\pi}{2}\right),\ k=0,1$$

$$=\sqrt{2}\left[\cos\left(\frac{2\pi}{3}+k\pi\right)+i\sin\left(\frac{2\pi}{3}+k\pi\right)\right],\ k=0,1$$

$$k=0 \Rightarrow z_3=\sqrt{2}\cdot\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

$$=\sqrt{2}\cdot\left(-\frac{1}{2}+i\cdot\frac{\sqrt{3}}{2}\right)=-\frac{\sqrt{2}}{2}+i\cdot\frac{\sqrt{6}}{2}$$

$$k=1 \Rightarrow z_4=\sqrt{2}\cdot\left[\cos\left(\frac{2\pi}{3}+\pi\right)+i\sin\left(\frac{2\pi}{3}+\pi\right)\right]$$

$$=-\sqrt{2}\left(-\frac{1}{2}+i\cdot\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{2}}{2}-i\cdot\frac{\sqrt{6}}{2}$$

$$z_{1,2,3,4}=\pm\frac{\sqrt{2}}{2}\pm\frac{\sqrt{6}}{2}$$

3.

$$A \sim \begin{bmatrix} \lambda & 2 & 3 & 8-\lambda \\ 0 & 7-\lambda & 1 & -7+\lambda \\ 0 & 0 & 7-\lambda & -7+\lambda \\ 0 & 0 & 0 & -7+\lambda \end{bmatrix}$$

1° $\lambda = 7$

$$A \sim \begin{bmatrix} 7 & 2 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{rang } A = 2$$

2° $\lambda = 0$

$$A \sim \begin{bmatrix} 0 & \boxed{2} & 3 & 8 \\ 0 & 7 & \boxed{1} & -7 \\ 0 & 0 & 7 & \boxed{-7} \\ 0 & 0 & 0 & -7 \end{bmatrix} \qquad \text{rang } A = 3$$

3° $\lambda \neq 7,\ \lambda \neq 0$ rang $A = 4$

4. $$\lim_{n\to\infty} \frac{(n+3)!}{(n+2)!+(n+4)!} = \lim_{n\to\infty} \frac{(n+2)!(n+3)}{(n+2)!+(n+2)!(n+3)(n+4)}$$

$$= \lim_{n\to\infty} \frac{(n+2)!(n+3)}{(n+2)!(1+n^2+4n+3n+12)} =$$

$$= \lim_{n\to\infty} \frac{n}{n^2} = 0$$

$$\lim_{n\to\infty} \left(\sqrt[4]{n^4+4n^3+4n^2+4n+1} - n\right) \cdot \frac{\sqrt[4]{n^4+4n^3+4n^2+4n+1} + n}{\sqrt[4]{n^4+\dots+1} + n}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^4+4n^3+4n^2+4n+1} - n^2}{\sqrt[4]{n^4+\dots+1} + n} \cdot \frac{\sqrt{n^4+\dots+1} + n^2}{\sqrt{n^4+\dots+1} + n^2}$$

$$= \lim_{n\to\infty} \frac{n^4+4n^3+4n^2+4n+1-n^4}{\left(\sqrt[4]{n^4+\dots+1} + n\right)\cdot\left(\sqrt{n^4+\dots+1} + n^2\right)}$$

$$= \lim_{n\to\infty} \frac{4n^3}{2n \cdot 2n^2} = 1$$